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Honors Geometry
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POINT-LINE DISTANCE FORMULA PROOF

given point (P, Q) and line $ax+by+c=0$

Formula: $d = \dfrac{|aP+bQ+c|}{\sqrt{a^2+b^2}}$

$ax+by+c=0$

$by=-ax-c$

$y = \dfrac{-a}{b}x - \dfrac{c}{b}$

slope of original line

$m_\perp = \dfrac{b}{a}$ and (P, Q)

so $\perp$ line is: $y - Q = \dfrac{b}{a}(x-P)$ point-slope form

OR

$y = \dfrac{b}{a}(x-p)+Q$

solve system of equations using 2 lines to find x

$$-\frac{a}{b}x - \frac{c}{b} = \frac{b}{a}(x-P)+Q$$

$$-\frac{ax}{b} - \frac{c}{b} = \frac{b(x-p)}{a} + Q$$

$\cdot a \longrightarrow$

$$-\frac{a^2x}{b} - \frac{ca}{b} = b(x-p)+Qa$$

$\cdot b \longrightarrow$

$$-a^2x - ca = b^2(x-p) + Qab$$

$$-a^2x - ac = b^2(x-p) + abQ$$

$-abQ \qquad -abQ$

$$-a^2x - ac - abQ = b^2(x-p)$$

$+a^2x \qquad +a^2x$

$$-ac-abQ = b^2(x-p)+a^2x$$

$$-ac-abQ = b^2x - b^2p + a^2x$$

$+b^2p \qquad +b^2p$

$$b^2p - ac - abQ = b^2x + a^2x$$

$$\frac{b^2p-ac-abQ}{b^2+a^2} = \frac{(b^2+a^2)(x)}{b^2+a^2}$$

x-coordinate of solution intersection $\longrightarrow$ $\dfrac{b^2p-ac-abQ}{b^2+a^2} = x$

GO ON

solve for y

$$x = \frac{b^2P - abQ - ac}{a^2 + b^2} \quad \& \quad by = -ax - c$$

$$by = \frac{-a}{1} \cdot \frac{b^2P - abQ - ac}{a^2 + b^2} - \frac{c}{1}$$

$$by = \frac{-a(b^2P - abQ - ac)}{a^2 + b^2} - \frac{c}{1}$$

$$by = \frac{-ab^2P + a^2bQ + a^2c}{a^2 + b^2} - \frac{c(a^2 + b^2)}{a^2 + b^2}$$

$$by = \frac{-ab^2P + a^2bQ + \cancel{a^2c} \cancel{- ca^2} - cb^2}{a^2 + b^2}$$

$$\frac{by}{b} = \frac{\frac{-ab^2P + a^2bQ - cb^2}{a^2 + b^2}}{b}$$

$$y = \frac{-ab^2P + a^2bQ - cb^2}{a^2 + b^2} \cdot \frac{1}{b}$$

$$y = \frac{\cancel{b}(-abP + a^2Q - cb)}{\cancel{b}(a^2 + b^2)}$$

y-coordinate of system solution

$$y = \frac{-abP + a^2Q - cb}{a^2 + b^2}$$

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using THE <u>distance formula</u> & solving

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{Given Point: } (P, Q)$$

$$d = \sqrt{\left(\frac{b^2P - abQ - ac}{a^2 + b^2} - \frac{P}{1}\right)^2 + \left(\frac{-abP + a^2Q - cb}{a^2 + b^2} - \frac{Q}{1}\right)^2}$$

$$d = \sqrt{\left(\frac{\cancel{b^2P} - abQ - ac - Pa^2 \cancel{- Pb^2}}{a^2 + b^2}\right)^2 + \left(\frac{-abP \cancel{+ a^2Q} - cb \cancel{- Qa^2} - Qb^2}{a^2 + b^2}\right)}$$

GO ON

(distance formula cont.)

$$d = \sqrt{\frac{(-abQ - ac - Pa^2)^2}{(a^2+b^2)^2} + \frac{(-abP - cb - Qb^2)^2}{(a^2+b^2)^2}}$$

$$d = \sqrt{\frac{(-abQ - ac - Pa^2)^2 + (-abP - cb - Qb^2)^2}{(a^2+b^2)^2}}$$

$$d = \sqrt{\frac{(-a)^2(bQ + c + aP)^2 + (-b)^2(aP + c + Qb)^2}{(a^2+b^2)^2}}$$

$$d = \sqrt{\frac{\cancel{(a^2+b^2)}(aP + bQ + c)^2}{\cancel{(a^2+b^2)}(a^2+b^2)}}$$

$$d = \sqrt{\frac{(aP + bQ + c)^2}{a^2+b^2}} = \frac{\sqrt{(aP+bQ+c)^2}}{\sqrt{a^2+b^2}}$$

$$d = \frac{\pm(aP + bQ + c)}{\sqrt{a^2+b^2}} = \frac{|aP + bQ + c|}{\sqrt{a^2+b^2}}$$

abs. value because distance can't be negative

yay!