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Honors Geometry
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POINT-LINE DISTANCE FORMULA PROOF

given point (P, Q) and line $ax + by + c = 0$

$$\text{Formula: } d = \frac{|aP + bQ + c|}{\sqrt{a^2 + b^2}}$$

$$ax + by + c = 0$$

$$by = -ax - c$$

$$y = \frac{-a}{b}x - \frac{c}{b}$$

slope of original line

$$m_{\perp} = \frac{b}{a} \text{ and } (P, Q)$$

$$\text{so } \perp \text{ line is: } y - Q = \frac{b}{a}(x - P)$$

point-slope form

OR

$$y = \frac{b}{a}(x - P) + Q$$

solve system of equations using 2 lines to find x

$$-\frac{a}{b}x - \frac{c}{b} = \frac{b}{a}(x - P) + Q$$

$$-\frac{ax}{b} - \frac{c}{b} = \frac{b(x - P)}{a} + Q$$

$$\cdot a \longrightarrow$$

$$-\frac{a^2x}{b} - \frac{ca}{b} = b(x - P) + Qa$$

$$\cdot b \longrightarrow$$

$$-a^2x - ca = b^2(x - P) + Qab$$

$$\begin{array}{r} -a^2x - ac = b^2(x - P) + abQ \\ -abQ \qquad \qquad -abQ \end{array}$$

$$\begin{array}{r} -a^2x - ac - abQ = b^2(x - P) \\ +a^2x \qquad \qquad \qquad +a^2x \end{array}$$

$$-ac - abQ = b^2(x - P) + a^2x$$

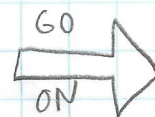
$$\begin{array}{r} -ac - abQ = b^2x - b^2P + a^2x \\ +b^2P \qquad \qquad \qquad +b^2P \end{array}$$

$$b^2P - ac - abQ = b^2x + a^2x$$

$$\frac{b^2P - ac - abQ}{b^2 + a^2} = \frac{(b^2 + a^2)x}{b^2 + a^2}$$

x -coordinate of solution intersection

$$\frac{b^2P - ac - abQ}{b^2 + a^2} = x$$

GO ON 

solve for y

$$x = \frac{b^2P - abQ - ac}{a^2 + b^2} \quad \& \quad by = -ax - c$$

$$by = \frac{-a}{1} \cdot \frac{b^2P - abQ - ac}{a^2 + b^2} - \frac{c}{1}$$

$$by = \frac{-a(b^2P - abQ - ac)}{a^2 + b^2} - \frac{c}{1}$$

$$by = \frac{-ab^2P + a^2bQ + a^2c}{a^2 + b^2} - \frac{c(a^2 + b^2)}{a^2 + b^2}$$

$$by = \frac{-ab^2P + a^2bQ + a^2c - ca^2 - cb^2}{a^2 + b^2}$$

$$\frac{by}{b} = \frac{-ab^2P + a^2bQ - cb^2}{a^2 + b^2}$$

$$y = \frac{-ab^2P + a^2bQ - cb^2}{a^2 + b^2} \cdot \frac{1}{b}$$

$$y = \frac{b(-abP + a^2Q - cb)}{b(a^2 + b^2)}$$

y-coordinate of system solution

$$y = \frac{-abP + a^2Q - cb}{a^2 + b^2}$$

using THE distance formula & solving

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given Point: (P, Q)

$$d = \sqrt{\left(\frac{b^2P - abQ - ac}{a^2 + b^2} - \frac{P}{1}\right)^2 + \left(\frac{-abP + a^2Q - cb}{a^2 + b^2} - \frac{Q}{1}\right)^2}$$

$$d = \sqrt{\left(\frac{b^2P - abQ - ac - Pa^2 - Pb^2}{a^2 + b^2}\right)^2 + \left(\frac{-abP + a^2Q - cb - Qa^2 - Qb^2}{a^2 + b^2}\right)^2}$$



(distance formula cont.)

$$d = \sqrt{\frac{(-abQ - ac - Pa^2)^2}{(a^2 + b^2)^2} + \frac{(-abP - cb - Qb^2)^2}{(a^2 + b^2)^2}}$$

$$d = \sqrt{\frac{(-abQ - ac - Pa^2)^2 + (-abP - cb - Qb^2)^2}{(a^2 + b^2)^2}}$$

$$d = \sqrt{\frac{(-a^2)(bQ + c + aP)^2 + (-b^2)(aP + c + Qb)^2}{(a^2 + b^2)^2}}$$

$$d = \sqrt{\frac{(a^2 + b^2)(aP + bQ + c)^2}{(a^2 + b^2)(a^2 + b^2)}}$$

$$d = \sqrt{\frac{(aP + bQ + c)^2}{a^2 + b^2}} = \frac{\sqrt{(aP + bQ + c)^2}}{\sqrt{a^2 + b^2}}$$

$$d = \frac{\pm(aP + bQ + c)}{\sqrt{a^2 + b^2}} = \frac{|aP + bQ + c|}{\sqrt{a^2 + b^2}}$$

abs. value
because distance
can't be negative

yay!